Letters

Elastic Three-Sphere Microswimmer in a Viscous Fluid

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We discuss the dynamics of a generalized three-sphere microswimmer in which the spheres are connected by two elastic springs. The natural length of each spring is assumed to undergo a prescribed cyclic change. We analytically obtain the average swimming velocity as a function of the frequency of cyclic change in the natural length. In the low-frequency region, the swimming velocity increases with frequency, and its expression reduces to that of the original three-sphere model by Najafi and Golestanian. Conversely, in the high-frequency region, the average velocity decreases with increasing frequency. Such behavior originates from the intrinsic spring relaxation dynamics of an elastic swimmer moving in a viscous fluid.

Microswimmers are tiny machines that swim in a fluid, such as sperm cells or motile bacteria, and they are expected to be applied to microfluidics and microsystems.[1] By transforming chemical energy into mechanical work, microswimmers change their shape and move in viscous environments. Over the length scale of microswimmers, the fluid forces acting on them are governed by the effect of viscous dissipation. According to Purcell's scallop theorem,[2] time-reversal body motion cannot be used for locomotion in a Newtonian fluid.[3] As one of the simplest models exhibiting broken time-reversal symmetry, Najafi and Golestanian proposed a three-sphere swimmer,[4,5] in which three in-line spheres are linked by two arms of varying length. This model is suitable for analytical analysis because it is sufficient to consider only the translational motion, and the tensorial structure of the fluid motion can be neglected. Recently, such a swimmer has been experimentally realized by using ferromagnetic particles at an air–water interface and by applying an oscillating magnetic field.[6]

The original Najafi–Golestanian model has been further extended to various situations, such as the case when one of the spheres has a larger radius[7] or when three spheres are arranged in a triangular configuration.[8] Montino and DeSimone considered the case in which one arm is periodically actuated while the other is replaced by a passive elastic spring.[9] It was shown that such a swimmer exhibits a delayed mechanical response of the passive spring with respect to the active arm. More recently, they analyzed the motion of a three-sphere swimmer with arms having active viscoelastic properties mimicking muscular contraction.[10] Recently, Nasouri et al. discussed the motion of an elastic two-sphere swimmer, in which one of the sphere is a neo-Hookean solid.[11]

Another approach for extending the Najafi–Golestanian model is to consider the arm motions as occurring stochastically,[12,13] rather than assuming a prescribed sequence of deformations.[4,5] In these models, the configuration space of a swimmer generally consists of a finite number of distinct states. A similar idea was employed by Sakaue et al., who discussed the propulsion of molecular machines or active proteins in the presence of hydrodynamic interactions.[14] Later, Huang et al. considered a modified three-sphere swimmer in a two-dimensional viscous fluid.[15] In their model, the spheres are connected by two springs, the lengths of which are assumed to depend on the discrete states that are cyclically switched. As a result, the dynamics of a swimmer consists of the spring relaxation processes, which follow after each switching event.

In this letter, we discuss a generalized three-sphere swimmer in which the spheres are simply connected by two harmonic springs. The main difference between this model and the previous models is that the natural length of each spring depends on time and is assumed to undergo a prescribed cyclic change. Whereas the arms in the Najafi–Golestanian model undergo a prescribed motion regardless of the force exerted by the fluid, the sphere motion in our model is determined by the natural spring lengths representing internal states of a swimmer and also by the force exerted by the fluid. In this sense, our model is more realistic to study the locomotion of active microswimmers. We analytically obtain the average swimming velocity as a function of the frequency of cyclic change in the natural length. In order to better illustrate our result, we first explain the case in which the two spring constants are identical and the two oscillation amplitudes of the natural lengths are the same. Then, we shall discuss a general case in which these quantities are different and the phase mismatch between the natural lengths is arbitrary.

The introduction of harmonic springs between the spheres leads to an intrinsic time scale of an elastic swimmer that characterizes its internal relaxation dynamics. When the frequency of cyclic change in the natural lengths is smaller than this characteristic time, the swimming velocity increases with frequency, as in the previous works.[5] In the high-frequency region, on the other hand, the motion of spheres cannot follow the change in the natural length, and the average swimming velocity decreases with increasing frequency. Such a situation resembles the dynamics of the Najafi–Golestanian three-sphere swimmer in a viscoelastic medium.[16] We also show that, owing to the elasticity that has been introduced, the proposed micromachine can swim even if the change in the natural lengths is reciprocal as long as its structural symmetry is violated. Although the considered swimmer appears to be somewhat trivial, it can be regarded as a generic model for microswimmers or protein machines since the behaviors of the previous models can be deduced from our model by taking different limits.

We generalize the Najafi–Golestanian three-sphere swimmer model to take into account the elasticity in the sphere motion. As schematically shown in Fig. 1, the present
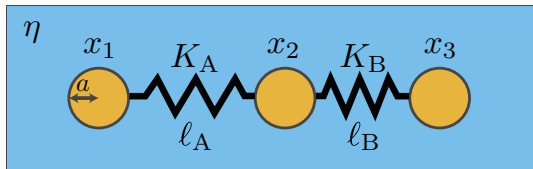
Fig. 1. (Color online) Elastic three-sphere microswimmer in a viscous fluid characterized by the shear viscosity η. Three identical spheres of radius a are connected by two harmonic springs with elastic constants K_A and K_B. The natural lengths of the springs, $\ell_A(t)$ and $\ell_B(t)$, depend on time and are assumed to undergo cyclic change [see Eqs. (6) and (7)]. The time-dependent positions of the spheres are denoted by $x_1(t)$, $x_2(t)$, and $x_3(t)$ in a one-dimensional coordinate system.

model consists of three hard spheres of the same radius a connected by two harmonic springs A and B with spring constants K_A and K_B, respectively. We assume that the natural lengths of these springs, denoted by $\ell_A(t)$ and $\ell_B(t)$, undergo cyclic time-dependent change. Their explicit time dependences will be specified later. The total energy of an elastic swimmer is then given by

$$E = \frac{K_A}{2}(x_2 - x_1 - \ell_A)^2 + \frac{K_B}{2}(x_3 - x_2 - \ell_B)^2, \quad (1)$$

where $x_i(t)$ ($i = 1, 2, 3$) are the positions of the three spheres in a one-dimensional coordinate system. We also assume $x_1 < x_2 < x_3$ without loss of generality. Owing to the hydrodynamic interaction, each sphere exerts a force on the viscous fluid of shear viscosity η and experiences an opposite force from it. In general, the surrounding medium can be viscoelastic,[16] but such an effect is not considered in this letter.

Denoting the velocity of each sphere by $\dot{x}_i$, we can write the equations of motion of the three spheres as

$$\dot{x}_1 = \frac{K_A}{6\pi\eta a}(x_2 - x_1 - \ell_A) - \frac{K_A}{4\pi\eta}\frac{(x_2 - x_1 - \ell_A)}{x_2 - x_1}$$
$$+ \frac{K_B}{4\pi\eta}\frac{(x_3 - x_2 - \ell_B)}{x_2 - x_1} - \frac{K_B}{4\pi\eta}\frac{(x_3 - x_2 - \ell_B)}{x_3 - x_1}, \quad (2)$$

$$\dot{x}_2 = \frac{K_A}{4\pi\eta}\frac{(x_2 - x_1 - \ell_A)}{x_2 - x_1} - \frac{K_A}{6\pi\eta a}(x_2 - x_1 - \ell_A)$$
$$+ \frac{K_B}{6\pi\eta a}(x_3 - x_2 - \ell_B) - \frac{K_B}{4\pi\eta}\frac{(x_3 - x_2 - \ell_B)}{x_3 - x_2}, \quad (3)$$

$$\dot{x}_3 = \frac{K_A}{4\pi\eta}\frac{(x_2 - x_1 - \ell_A)}{x_3 - x_1} - \frac{K_A}{4\pi\eta}\frac{(x_2 - x_1 - \ell_A)}{x_3 - x_2}$$
$$+ \frac{K_B}{4\pi\eta}\frac{(x_3 - x_2 - \ell_B)}{x_3 - x_2} - \frac{K_B}{6\pi\eta a}(x_3 - x_2 - \ell_B), \quad (4)$$

where we have used the Stokes' law for a sphere and the Oseen tensor in a three-dimensional viscous fluid. The swimming velocity of the whole object can be obtained by averaging the velocities of the three spheres:

$$V = \frac{1}{3}(\dot{x}_1 + \dot{x}_2 + \dot{x}_3). \quad (5)$$

One of the advantages of the present formulation is that the motion of the spheres is simply described by coupled ordinary differential equations. Moreover, the force-free condition for the whole system[4,5] is automatically satisfied in the above equations.

Next, we assume that the two natural lengths of the springs undergo the following periodic changes:

$$\ell_A(t) = \ell + d_A \cos(\Omega t), \quad (6)$$

$$\ell_B(t) = \ell + d_B \cos(\Omega t - \phi). \quad (7)$$

In the above, ℓ is the common constant length, d_A and d_B are the amplitudes of the oscillatory change, Ω is the common frequency, and ϕ is the mismatch in phase between the two cyclic changes. The time-reversal symmetry of the spring dynamics exists only when $\phi = 0$ or π; otherwise, the time-reversal symmetry is broken. In the following analysis, we generally assume that $d_A, d_B, a \ll \ell$ and focus on the leading-order contribution. It is convenient to introduce a characteristic time scale $\tau = 6\pi\eta a/K_A$. Then we use ℓ to scale all the relevant lengths (x_i, a, d_A, and d_B) and employ τ to scale the frequency, i.e., $\hat{\Omega} = \Omega\tau$. By further defining the ratio between the two spring constants as $\lambda = K_B/K_A$, the coupled Eqs. (2)–(4) can be made dimensionless.

In order to discuss the essential outcome of the present model, we shall first consider the simplest symmetric case, i.e., $\lambda = 1$, $d_A = d_B = d$, and $\phi = \pi/2$. Hence, Eq. (7) now reads $\ell_B(t) = \ell + d \sin(\Omega t)$. For our later calculation, it is useful to introduce the following spring lengths with respect to ℓ:

$$u_A = x_2 - x_1 - \ell, \quad u_B = x_3 - x_2 - \ell. \quad (8)$$

Notice that these quantities are related to the sphere velocities in Eqs. (2)–(4) as

$$\dot{u}_A = \dot{x}_2 - \dot{x}_1, \quad \dot{u}_B = \dot{x}_3 - \dot{x}_2. \quad (9)$$

Using Eqs. (2)–(4) and solving Eq. (9) in the frequency domain, we obtain the following expressions after inverse Fourier transform

$$u_A(t) \approx \frac{9 - 3\hat{\Omega} + 5\hat{\Omega}^2 + \hat{\Omega}^3}{9 + 10\hat{\Omega}^2 + \hat{\Omega}^4} d \cos(\Omega t)$$
$$+ \frac{6\hat{\Omega} - 4\hat{\Omega}^2 + 2\hat{\Omega}^3}{9 + 10\hat{\Omega}^2 + \hat{\Omega}^4} d \sin(\Omega t), \quad (10)$$

$$u_B(t) \approx -\frac{6\hat{\Omega} + 4\hat{\Omega}^2 + 2\hat{\Omega}^3}{9 + 10\hat{\Omega}^2 + \hat{\Omega}^4} d \cos(\Omega t)$$
$$+ \frac{9 + 3\hat{\Omega} + 5\hat{\Omega}^2 - \hat{\Omega}^3}{9 + 10\hat{\Omega}^2 + \hat{\Omega}^4} d \sin(\Omega t), \quad (11)$$

where we have used $a/\ell \ll 1$.

According to the calculation by Golestanian and Ajdari,[5] the average swimming velocity of a three-sphere swimmer can generally be expressed up to the leading order in u_A/ℓ and u_B/ℓ as

$$\overline{V} = \frac{7a}{24\ell^2}\langle u_A \dot{u}_B - \dot{u}_A u_B \rangle, \quad (12)$$

where the averaging $\langle \cdots \rangle$ is performed by time integration in a full cycle. The above expression indicates that the average velocity is determined by the area enclosed by the orbit of periodic motion in the configuration space.[5] Using Eqs. (10) and (11) for an elastic microswimmer with $d/\ell, a/\ell \ll 1$, we obtain the lowest-order contribution as

$$\overline{V} = \frac{7d^2 a}{24\ell^2\tau}\frac{3\hat{\Omega}(3 + \hat{\Omega}^2)}{9 + 10\hat{\Omega}^2 + \hat{\Omega}^4}, \quad (13)$$

which is an important result of this letter.

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We first consider the small-frequency limit of $\hat{\Omega} \ll 1$. Physically, this limit corresponds to the case when the spring constant K_A is very large. We easily obtain

$$u_A(t) \approx d\cos(\Omega t), \quad u_B(t) \approx d\sin(\Omega t), \qquad (14)$$

and

$$\overline{V} \approx \frac{7d^2 a\Omega}{24\ell^2}, \qquad (15)$$

which exactly coincides with the average velocity of the Najafi–Golestanian swimmer with identical spheres.[4,5] This is reasonable because the two spring lengths u_A and u_B are in phase with their respective natural lengths ℓ_A and ℓ_B, as we can see from Eqs. (6), (7), and (14). Notice that the average velocity increases as $\overline{V} \sim \Omega$ in this limit, while it does not depend on the fluid viscosity η.[4,5]

In the opposite large-frequency limit of $\hat{\Omega} \gg 1$, on the other hand, we have

$$u_A(t) \approx \frac{\sqrt{5}d}{\Omega\tau}\cos[\Omega t - \arctan 2], \qquad (16)$$

$$u_B(t) \approx \frac{\sqrt{5}d}{\Omega\tau}\sin[\Omega t - (\pi - \arctan 2)], \qquad (17)$$

where $\arctan 2 \approx 1.107$ and

$$\overline{V} \approx \frac{21d^2 a}{24\ell^2\Omega\tau^2}. \qquad (18)$$

We see here that u_A and u_B are out of phase with respect to the natural lengths ℓ_A and ℓ_B, while the average velocity decreases as $\overline{V} \sim \Omega^{-1}$ when Ω is increased. When the spring constant K_A is small, it takes time for a spring to relax to its natural length, which leads to a delay in the mechanical response. The crossover frequency between the above two regimes is determined by $\hat{\Omega}^* \sim 1$. The general frequency dependence of Eq. (13) is shown in Fig. 2(a) for $\lambda = 1$ (black line). It shows a maximum around $\hat{\Omega} \sim 1$, as expected.

Recently, we investigated the motion of the Najafi–Golestanian three-sphere swimmer in a viscoelastic medium.[16] We derived a relation that connects the average swimming velocity and the frequency-dependent viscosity of the surrounding medium. In this relation, the viscous contribution can exist only when the time-reversal symmetry is broken, whereas the elastic contribution is present only when the structural symmetry of the swimmer is broken. In particular, we calculated the average swimming velocity when the surrounding viscoelastic medium is described by a simple Maxwell fluid with a characteristic time scale τ_M. It was shown that the viscous term increases as $\overline{V} \sim \Omega$ for $\Omega\tau_M \ll 1$, while it decreases as $\overline{V} \sim \Omega^{-1}$ for $\Omega\tau_M \gg 1$. This is a unique feature of a swimmer in a viscoelastic medium,[16–18] and such a reduction occurs simply because the medium responds elastically in the high-frequency regime. We note that the frequency dependence of $\overline{V}$ for an elastic three-sphere swimmer, as obtained in Eq. (13), is analogous to that for the Najafi–Golestanian swimmer in a viscoelastic Maxwell fluid. In other words, an elastic microswimmer in a viscous fluid exhibits "viscoelastic" effects as a whole.

Having discussed the simplest situation of the proposed elastic swimmer, we now show the result for a general case when $K_A \neq K_B$ (or $\lambda \neq 1$), $d_A \neq d_B$, and the phase mismatch
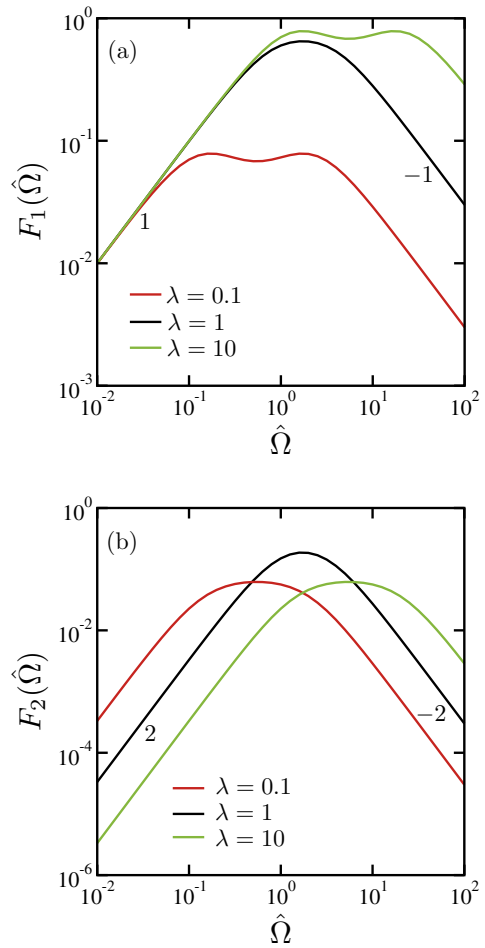

Fig. 2. (Color online) Plots of the scaling functions (a) $F_1(\hat{\Omega}; \lambda)$ and (b) $F_2(\hat{\Omega}; \lambda)$ defined in Eqs. (22) and (23), respectively, as functions of $\hat{\Omega} = \Omega\tau$ for $\lambda = K_B/K_A = 0.1$, 1, and 10. The numbers indicate the slope representing the exponent of the power-law behaviors.

ϕ in Eq. (7) is arbitrary. By repeating the same calculation as before, the spring lengths in Eq. (8) now become

$$u_A(t) \approx \frac{1}{9\lambda^2 + 2(2 + \lambda + 2\lambda^2)\hat{\Omega}^2 + \hat{\Omega}^4}$$
$$\times \Big\{[9\lambda^2 + (4 + \lambda)\hat{\Omega}^2]d_A\cos(\Omega t)$$
$$+ 2(3\lambda^2 + \hat{\Omega}^2)\hat{\Omega}d_A\sin(\Omega t)$$
$$- 2\lambda(1 + \lambda)\hat{\Omega}^2 d_B\cos(\Omega t - \phi)$$
$$- \lambda(-3\lambda + \hat{\Omega}^2)\hat{\Omega}d_B\sin(\Omega t - \phi)\Big\}, \qquad (19)$$

$$u_B(t) \approx \frac{1}{9\lambda^2 + 2(2 + \lambda + 2\lambda^2)\hat{\Omega}^2 + \hat{\Omega}^4}$$
$$\times \Big\{-2(1 + \lambda)\hat{\Omega}^2 d_A\cos(\Omega t)$$
$$+ (3\lambda - \hat{\Omega}^2)\hat{\Omega}d_A\sin(\Omega t)$$
$$+ \lambda[9\lambda + (1 + 4\lambda)\hat{\Omega}^2]d_B\cos(\Omega t - \phi)$$
$$+ 2\lambda(3 + \hat{\Omega}^2)\hat{\Omega}d_B\sin(\Omega t - \phi)\Big\}, \qquad (20)$$

respectively, where we have used $a/\ell \ll 1$. Using Eq. (12) again, we finally obtain the lowest-order general expression of the average velocity as

$$\overline{V} = \frac{7 d_A d_B a}{24 \ell^2 \tau} F_1(\hat{\Omega}; \lambda) \sin\phi$$
$$- \frac{7(\lambda - 1) d_A d_B a}{12 \ell^2 \tau} F_2(\hat{\Omega}; \lambda) \cos\phi$$
$$+ \frac{7(d_A^2 - d_B^2 \lambda) a}{24 \ell^2 \tau} F_2(\hat{\Omega}; \lambda), \tag{21}$$

where the two scaling functions are defined by

$$F_1(\hat{\Omega}; \lambda) = \frac{3\lambda \hat{\Omega}(3\lambda + \hat{\Omega}^2)}{9\lambda^2 + 2(2 + \lambda + 2\lambda^2)\hat{\Omega}^2 + \hat{\Omega}^4}, \tag{22}$$

$$F_2(\hat{\Omega}; \lambda) = \frac{3\lambda \hat{\Omega}^2}{9\lambda^2 + 2(2 + \lambda + 2\lambda^2)\hat{\Omega}^2 + \hat{\Omega}^4}. \tag{23}$$

In Fig. 2, we plot the above scaling functions as functions of $\hat{\Omega}$ for $\lambda = 0.1$ and 10. Notice, however, that these two cases are essentially equivalent because we can always exchange the springs A and B, whereas we have defined the relaxation time τ by using K_A.

When $\lambda = 1$, $d_A = d_B$, and $\phi = \pi/2$, only the first term remains, and Eq. (21) reduces to Eq. (13), as expected. When $\lambda \neq 1$, on the other hand, the second term is present even if $\phi = 0$. The third term is also present when $d_A^2 \neq d_B^2 \lambda$, regardless of the phase mismatch ϕ. Notice that the first term represents the broken time-reversal symmetry for $\phi \neq 0$, whereas both the second and third terms reflect the structural asymmetry of an elastic three-sphere swimmer.[16] To be more precise, the second term is due to the difference between the relaxation times of the two springs, and the third term reflects the asymmetric changes of their natural lengths.

It is interesting to note that the frequency dependence of the second and third terms in Eq. (21), represented by $F_2(\hat{\Omega}, \lambda)$, is different from that of the first term, represented by $F_1(\hat{\Omega}, \lambda)$. According to Eq. (23), $\overline{V}$ due to the second and third terms increases as $\overline{V} \sim \Omega^2$ for $\hat{\Omega} \ll 1$, whereas it decreases as $\overline{V} \sim \Omega^{-2}$ for $\hat{\Omega} \gg 1$. In general, the overall swimming velocity depends on various structural parameters and exhibits a complex frequency dependence. For example, $F_1(\hat{\Omega}, \lambda)$ in Fig. 2(a) exhibits a non-monotonic frequency dependence (two maxima) for $\lambda = 0.1$ or 10 (namely, when $\lambda \neq 1$). On the other hand, an important common feature in all the terms in Eq. (21) is that $\overline{V}$ decreases for $\hat{\Omega} \geq 1$, which is characteristic of elastic swimmers.

We confirm again that Eq. (21) reduces to the result by Golestanian and Ajdari,[5] i.e., $\overline{V} = 7 d_A d_B a\Omega \sin\phi/(24\ell^2)$, when the two spring constants are infinitely large, i.e., $K_A, K_B \to \infty$ and $\lambda = 1$. The third term in Eq. (21) vanishes even if $d_A \neq d_B$ because $\hat{\Omega} \to 0$ holds in this limit. In the modified three-sphere swimmer model considered by Montino and DeSimone, one of the two arms was replaced by a passive elastic spring.[9] Their model can be obtained from the present model simply by setting one of the spring constants to be infinitely large, say $K_A \to \infty$, and by regarding the natural length of the other spring as a constant, say $\ell_B = \ell$ (or $d_B = 0$). The continuous changes of the natural lengths introduced in Eqs. (6) and (7) are a straightforward generalization of cyclically switched discrete states considered in the previous studies.[12–15] We finally note that a model similar to the present one was considered in Ref. 19, although that study focused only on the low-frequency region and did not discuss the entire frequency dependence. Using coupled Langevin equations, the authors of Ref. 19 mainly investigated the interplay between self-driven motion and diffusive behavior,[19] which is also an important aspect of microswimmers.

To summarize, we have discussed the locomotion of a generalized three-sphere microswimmer in which the spheres are connected by two elastic springs and the natural length of each spring is assumed to undergo a prescribed cyclic change. As shown in Eqs. (13) and (21), we have analytically obtained the average swimming velocity $\overline{V}$ as a function of the frequency Ω of cyclic change in the natural length. In the low-frequency region, the swimming velocity increases with frequency and reduces to the original three-sphere model by Najafi and Golestanian.[4,5] Conversely, in the high-frequency region, the velocity decreases with increasing frequency. This property reflects the intrinsic spring relaxation dynamics of an elastic swimmer in a viscous fluid.

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Note added in proof—After this letter was accepted, we became aware that a similar model was considered in [J. Pande and A.-S. Smith, Soft Matter **11**, 2364 (2015)] and [J. Pande et al., New J. Phys. **19**, 053024 (2017)]. Although the three spheres are driven by known forces in their model, the frequency dependence of the average velocity is the same as our result. We thank the authors of these articles for pointing out their works.