J. Phys. France **51** (1990) 2395–2398 1[er] NOVEMBRE **1990**, PAGE 2395

Classification
Physics Abstracts
05.40 — 36.20 — 68.10

Short Communication

Spectral dimension of fluid membranes

Shigeyuki Komura(*) and Artur Baumgärtner

Institut für Festkörperforschung, Forschungszentrum Jülich, D-5170 Jülich, F.R.G.

(Received 13 August 1990, accepted 20 August 1990)

Abstract. — The spectral dimension d_s of polymerized and fluid self-avoiding vesicles are investigated by Monte Carlo methods. For both cases we obtained d_s=2, which indicates that these surfaces belong to the same class of "microcanonical" surfaces.

Properties of flexible sheet polymer networks have been explored in recent theoretical investigations [1] . Polymerized membranes have a nonzero shear modulus in the plane and are said to be solid-like, whereas fluid membranes are not rigid in the plane and have zero in-plane modulus.

It has been shown using computer simulations [2, 3] that polymerized self-avoiding membranes are essentially flat and the corresponding in-plane squared radius of gyration is proportional to the number of monomers on the surface,

$$R_{\parallel}^2 \sim N^{\nu} \text{ with } \nu \approx 1.0.$$

Very recently, a model of *fluid* self-avoiding membranes has been proposed which exhibits, in contrast to polymerized membranes, crumpled shapes with $\nu \approx 0.8$ [3] . One important question with respect to the differences between polymerized and fluid membranes is related to their internal connectivity which is commonly characterised by the spectral dimension d_s [4, 5] . The spectral dimensions of various classes of membranes have been discussed in detail by Cates [6] . For the case of polymerized surfaces with random connectivity, it is expected $d_s = 2$. Very recently, it has been shown that even for fractal surfaces with hierarchical connectivity [7] the spectral dimension is, somewhat unexpectedly, *not* changed and also $d_s = 2$.

In the present Communication, we report on Monte Carlo studies of the spectral dimension for *fluid* membranes. For comparison, we also include the results of d_s for the corresponding model of polymerized membranes.

The standard way to measure the spectral dimension is related to a random walk on the given surface [4, 5] . Given a particular realization of the membrane, the corresponding mean-square

(*) *Address after 1 April 1991:* Institute of Physics, College of Arts and Sciences, University of Tokyo, Meguro-Ku, Tokyo 153, Japan.

displacements at time t of a random walk on the surface is expressed as

$$\langle r^2(t)\rangle \sim t^{d_s/d_f} \tag{1}$$

where $d_f = 2/\nu$ is the fractal dimension of the model membrane. In practice, $r^2(t)$ has to be averaged over various walks on one particular frozen realization of the surface as well over different realizations. For convenience, we used a spherically closed surface ("vesicle") in order to take into account the periodic boundary conditions for the random walk properly.

The simulation technique for generating various conformations of self-avoiding vesicles is the same as has been used previously [3]. In a polymerized vesicle, the connectivity at each monomer is fixed. For a fluid vesicle, however, we relax the restriction on the fixed connectivity; we allow the monomers to exchange their neighbors, but keeping the rule that the topology and the integrity of the stucture should be preserved ("triangulation" procedure). This procedure provides for a given monomer to escape after several bond exchanges from its original neighborhoods of monomers, and hence represents a "fluid" particle.

The averaged mean-square displacement of a random walker on various sizes of vesicles are analyzed by using the crossover scaling form (similar to that of diffusion on percolating clusters, e.g., Ref.[8]),

$$\langle r^2(t)\rangle = N^{\nu} f\left(\frac{t}{N^{2/d_s}}\right), \tag{2}$$

where $f(x)$ is a scaling function with $f(x) \sim x^{d_s/d_f}$ for $x \ll 1$ and $f(x) =$ const. for $x \gg 1$. With this scaling function, the mean-square displacement behaves as $\langle r^2(t)\rangle \sim t^{d_s/d_f}$ for $t \ll \tau$, and $\langle r^2(t)\rangle \sim N^{\nu}$ for $t \gg \tau$, where $\tau = N^{2/d_s}$ gives the crossover time. For very long times ($t \gg \tau$), $\langle r^2(t)\rangle$ saturates and remains constant reflecting the fact that the displacement of the random walker are bounded by the finite size of the vesicles. The exponent ν has been estimated according to the relation (2) at $x \gg 1$, i.e., $\langle r^2(t)\rangle \sim N^{\nu}$, which yields $\nu = 0.98 \pm 0.02$ and 0.83 ± 0.02 for polymerized and fluid vesicles, respectively. Since we expect that under the scaling of (2) all data should collapse to a single curve, we have estimated the spectral dimension d_s from several attempts to obtain optimal overlap of the curves for all N. This is presented in figure 1, where $y = \langle r^2(t)\rangle / N^{\nu}$ is shown as a function of $x = t/N^{2/d_s}$. Fixing ν =0.98 and 0.83 for the two types of vesicles, we obtained the best fit using $d_s = 1.96 \pm 0.05$ for polymerized and d_s= 2.02 $\pm$ 0.04 for fluid vesicles. Of course, these estimates of $\nu = 2/d_f$ and d_s are consistent with the slope $f(x) \sim x^{d_s/d_f}$ for $x \ll 1$. Our conjecture is that $d_s = 2$ for both polymerized and fluid vesicles.

It is worthwhile to point out the next consideration on the crossover time τ. The return probability $P(t)$ that the random walker comes back to the starting point at time t scales as [4,5]

$$P(t) \sim \frac{1}{\langle r^2(t)\rangle^{d_f/2}} \sim \frac{1}{t^{d_s/2}}. \tag{3}$$

Therefore the number of accessible sites $\Sigma(t)$ increases as $\Sigma(t) \sim (P(t))^{-1} \sim t^{d_s/2}$. When t comes close to the crossover time τ, $t \sim N^{2/d_s}$ holds. This means that the number of accessible sites amounts to the order of $\Sigma\ (t) \sim t^{d_s/2} \sim N$. This can be interpreted that once the random walker has visited all N sites, it will start to access the sites it has already visited and therefore the mean-square displacement begins to saturate for $t > \tau$.

Finally, it should be noted that $d_s = 2$ of the present model for fluid membranes implies that this type of model belongs to the class of "microcanonical" surfaces [6] , which is commonly restricted

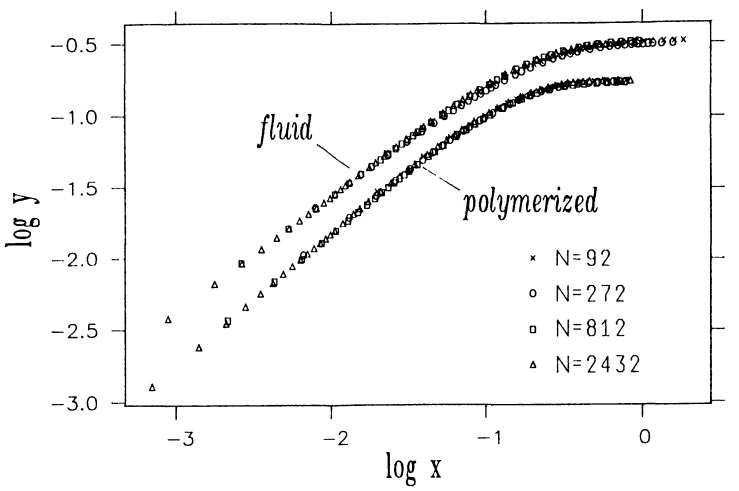

Fig. 1. — Scaling plot for average mean-squared displacement of a random walker on polymerized and fluid self-avoiding vesicles. Here $x = t/N^{2/d_s}$ and $y = \langle r^2(t) \rangle / N^{\nu}$ with $d_s = 1.96$ and $\nu = 0.98$ for polymerized vesicles $d_s = 2.02$ and $\nu = 0.83$ for fluid vesicles.

to polymerized random surfaces [2, 3, 9] subjected to the constraints of constant surface area, $S =$ const., and fixed "local" connectivity. With respect to the present results for fluid membranes, $d_f = 2.5$, it seems to be reasonable to extend the class of "microcanonical" surfaces by including fluid surfaces with $S =$ const., but with the weaker constraint of fixed "global" connectivity, or in other words, constant numbers of vertices, faces and edges. Moreover, with respect to the recent findings of $d_s = 2$ for deterministic fractal surfaces with $d_f = 2.33$ [7] , it is suggestive to attribute $d_s = 2$ to the class of "microcanonical" manifolds in general, including (so far as we know currently) random polymerized, fluid and fractal surfaces.

Acknowledgements.

One of us (S.K.) is very grateful for the hospitality of IFF at Forschungszentrum Jülich. He also would like to express his appreciation to Dr. M.E. Cates, Dr. D. Kroll and Dr. Y. Taguchi for useful discussions. This work was done on CRAY X-MP/416.

References

[1] See for instance Proceedings of the Fifth Jerusalem Winter School, Statistical Mechanics of Membranes and Surfaces, Eds. D.R. Nelson, T. Piran and S. Weinberg (World Scientific) 1989.

[2] PLISCHKE M. and BOAL D., *Phys. Rev. A* **38** (1988) 4943;
ABRAHAM F.F., RUDGE W.E. and PLISCHKE M., *Phys. Rev. Lett.* **62** (1989) 1757;
HO J.-S. and BAUMGÄRTNER A., *Phys. Rev. Lett.* **63** (1989) 1324.

[3] BAUMGÄRTNER A. and HO J.-S., *Phys. Rev. A* **41** (1990) 5747;
HO J.-S. and BAUMGÄRTNER A., *Europhys. lett.* **12** (1990) 295.

[4] ALEXANDER S. and ORBACH R., *J. Phys. Lett. France* **43** (1982) 625.
[5] RAMMAL R. and TOULOUSE G., *J. Phys. Lett. France* **44** (1983) 13.
[6] CATES M.E., *Phys. Rev. Lett.* **53** (1984) 926;
CATES M.E., *J. Phys. France* **46** (1985) 1059;
CATES M.E., *Phys. Lett. B* **161** (1985) 363.
[7] GIUGLIARELLI G., MARTIN A. and STELLA A.L., *Europhys. Lett.* **11** (1990) 101.
[8] GEFEN Y., AHARONY A. and ALEXANDER S., *Phys. Rev. Lett.* **50** (1983) 77.
[9] GROSS D.J., *Phys. Lett.* **138B** (1984) 185;
BILLOIRE A., GROSS D.J. and MARINARI E., *Phys. Lett.* **139B** (1984) 75;
DUPLANTIER B., *Phys. Lett.* **141B** (1984) 239.

Cet article a été imprimé avec le Macro Package "Editions de Physique Avril 1990".